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Technical Comments

Comment on "Flow Near the Trailing Edge of an Airfoil"

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AFTER publication of Ref. 1, the author's attention has been drawn to the existence of Refs. 2 and 3, which contain results closely related to some of the points discussed in Ref. 1; due acknowledgment is made hereby. Some further developments of the subject, prompted by the cited references, will be discussed hereafter.

Using the notations of Ref. 1, we consider an airfoil contour, defined in the complex plane z by a conformal mapping function $z=f(\zeta)$, which carries it onto the unit circle in the plane ζ; the tangent angles and radii of curvature in the two planes will be denoted by $\bar{t}$ and $\bar{\tau}$, and $\bar{\rho}$, respectively. The flow potential $F(z)$ can be differentiated to get the complex velocity: $w=\mathrm{d}F/\mathrm{d}z=u-iv=|w|\exp(-i\bar{t})$ and its derivative: $\mathrm{d}w/\mathrm{d}z=\mathrm{d}^2F/\mathrm{d}z^2$; we shall employ the tilde to signify the conjugate of a complex variable. Bernoulli's theorem reading

$$p+\rho/2\, w\times\tilde{w} = \text{const}$$

one may differentiate along an arbitrary direction $\mathrm{d}z = \mathrm{d}\sigma \exp(i\gamma)$, to get

$$\frac{\mathrm{d}p}{\mathrm{d}\sigma} = -\rho/2 \exp(i\gamma)\left[w\frac{\mathrm{d}\tilde{w}}{\mathrm{d}z}+\tilde{w}\frac{\mathrm{d}w}{\mathrm{d}z}\right] \tag{1}$$

After some manipulations, one obtains

$$w\frac{\mathrm{d}\tilde{w}}{\mathrm{d}z} = \exp(-i\gamma)\times P; \qquad \tilde{w}\frac{\mathrm{d}w}{\mathrm{d}z} = \exp(-i\gamma)\times\tilde{P} \tag{2}$$

with

$$P = \frac{\mathrm{d}(|w|^2/2)}{\mathrm{d}\sigma}+i|w|^2\frac{\mathrm{d}\bar{t}}{\mathrm{d}\sigma} \tag{3}$$

and finally

$$\frac{\mathrm{d}p}{\mathrm{d}\sigma} = -\frac{\rho}{2}(P+\tilde{P}) = -\rho\,\mathrm{real}\,(P)$$
$$= -\rho\,\mathrm{real}\left[\exp(i\gamma)\times w\times\frac{\mathrm{d}\tilde{w}}{\mathrm{d}z}\right] \tag{4}$$

This formula generalizes Eq. (5) of Ref. 1 and gives the pressure gradient along any direction. In particular, along a streamline (or the airfoil contour), $\gamma = \bar{t}$ and $\sigma = s$; then, P becomes

$$P = \frac{\mathrm{d}(|w|^2/2)}{\mathrm{d}s}+i\frac{|w|^2}{\bar{r}} \tag{5}$$

(since $\mathrm{d}\bar{t}/\mathrm{d}s = 1/\bar{r}$, by Frénet's theorem). P may be regarded as a sort of complex pressure gradient (see Ref. 2); but it would be wrong if one were to identify its real and imaginary parts with the tangential and normal pressure derivatives. Only the real part of P has a definite physical meaning: it coincides with the actual pressure gradient, whatever the direction of differentiation [Eq. (1)]. However, both components of P should be taken into consideration when one undertakes an analysis of the flow singularities at the trailing edge (TE). For a thorough discussion of the implications of a possible singularity in the tangential pressure gradient, the reader is referred to Refs. 2 and 3; as for the influence of the streamline curvature, $1/\bar{r}_s$, in the following we shall develop further the arguments put forward in Ref. 1.

Along the airfoil surface, $\bar{r}$ is a geometrical characteristic of the contour and is, usually, finite (with some exceptions, which will be taken up later); Eq. (4) is, however, also valid along the streamline emerging from the TE. But, as proved in Ref. 1, the curvature of this streamline tends to infinity when approaching the TE from downstream, unless the angle of attack of the airfoil has a certain singular value, α^*, which is a proper feature of each particular contour (see also Ref. 2). This curvature discontinuity imposes a sudden change of direction to the velocity vector, which cannot, usually, be negotiated by the fluid without creating a small separated region just upstream of the TE, even in the case of a cusped airfoil. However, this upstream influence, extending to the airfoil surface proper, of the streamline curvature discontinuity cannot be accounted for by a standard boundary-layer (BL) approach (since the BL equations are parabolic); a strong viscous-inviscid coupling near the TE actually comes into play, which may alter significantly the flow around the whole airfoil, reducing the lift slope and increasing the drag.

Further thoughts on these matters have led us to put forward, in Ref. 1, Proposition IX, that in a real flow, the interplay between the boundary layer and the outer potential motion is such that, when the angle of attack α is varied, the shape of the streamlines enclosing the airfoil and the BL will form an equivalent airfoil contour whose own singular angle of attack α^* (which renders the streamline curvature finite at the TE) would coincide with the current value of α. This self-adaptation often implies excessive local thickening of the BL, inducing a certain amount of separation. It would, therefore, be beneficial if the shape of the solid airfoil contour, near the TE, were such as to permit the adaptation without separation, at least at the cruise incidence. Some hints toward this goal are provided hereafter, by considering a particular airfoil shape, for which analytic formulas can be deduced.

We recall that the curvature of a simple Joukowski contour (i.e., one derived from a circle by the transform $z=\zeta+1/\zeta$), is infinite at the TE. The TE being a singular point itself for any airfoil, all derivatives there have to be computed by one-sided differentiation. This does not forbid, nevertheless, the curvature of the emerging streamline to be finite at the α^* peculiar to the airfoil in question. However, there does exist a particular Joukowski airfoil for which $1/\bar{r}$ remains finite; this is the circular-arc airfoil discussed in Ref. 1 (in the notations of Ref. 3, the radius R of the generating circle is equal to $1/\cos\beta$). The value of α^*, which renders the streamline curvature at the TE finite for this airfoil, is negative and can be computed using Eq. (16) of Ref. 1, to get

$$\alpha^* \tan^{-1}(-6a/c) \tag{6}$$

where c is the airfoil chord, and a the arc bow [there is a numerical error in Eq. (17) of Ref. 1]. We note that α^*, throughout this discussion, is referred to the zero-lift incidence, α_0, of the airfoil being considered; in our case, $\alpha_0 = \tan^{-1}(-2a/c)$. Computation of the actual pressure distribution shows that, effectively, all singularities are removed, and the streamline emerges as a prolongation

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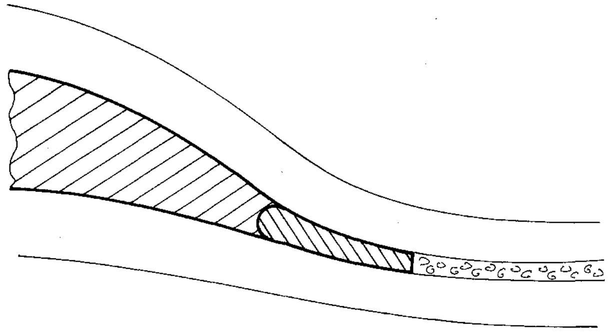

Fig. 1 Sketch of a favorable trailing-edge shape.

of the airfoil contour. Actually, all loading vanishes at the TE for this incidence.

All of this information makes us believe that, for a real airfoil, the TE shape should resemble that of our example: i.e., be cusped (albeit of finite thickness, see the discussion in Ref. 1) and have a certain amount of negative camber, locally (Fig. 1). We note that a certain curvature at the TE is always required for a lift-carrying airfoil. Moreover, to satisfy the requirement of a finite streamwise pressure gradient, the airfoil contour would have to be shaped in such a way as to ensure compliance with the condition real(P) = finite [see Eq. (4)] at the TE for $\alpha = \alpha^*$. Full adaptation, in the sense of the previously cited proposition, without formation of a separation bubble would be provided, of course, only at the proper α^* of the equivalent airfoil (drawn over the BL thickness). However, the flow might be tolerant and, in addition, a small moving tab, as suggested in Ref. 1, could widen the adaptation range of incidences. As pointed out in Ref. 3, removal of loading at the TE proper does not imply giving up the advantages of aft loading altogether; as a bonus, the proposed shape should display a lower negative pitching moment, thus reducing the trim drag of the aircraft in cruise.

Certainly, experimental verification of ideas put forward here (and in Refs. 1–3) would be welcome, but rather tricky.

References

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